Market VaR Estimation Methods

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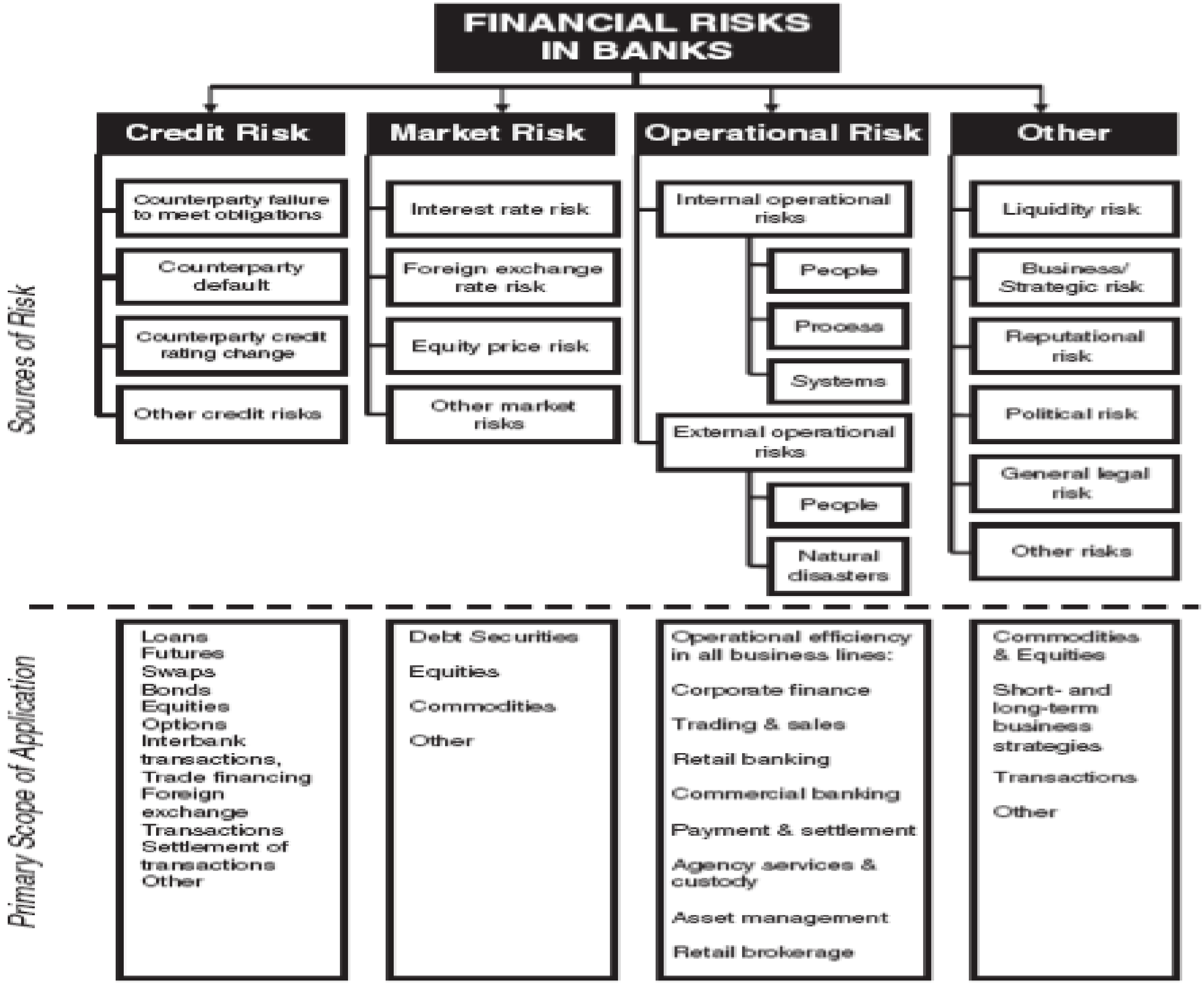
Market Risk

The risk of losses (in on- and off-balance sheet positions) arising from movements in market prices, including interest rates, exchange rates, equity values, commodity prices.

- Major components: interest rate risk, equity position risk, foreign exchange risk, and commodity risk.


FINANCIAL RISKS IN BANKS
Credit Risk
Market Risk
Operational Risk
Other
Sources of Risk
Counterparty failure to meet obligations
Counterparty default
Counterparty credit rating change
Other credit risks
Interest rate risk
Foreign exchange rate risk
Equity price risk
Other market risks
Internal operational risks
People
Process
Systems
External operational risks
People
Natural disasters
Liquidity risk
Business/ Strategic risk
Reputational risk
Political risk
General legal risk
Other risks
Primary Scope of Application
Loans
Futures
Swaps
Bonds
Equities
Options
Interbank transactions,
Trade financing
Foreign exchange
Transactions
Settlement of transactions
Other
Debt Securities
Equities
Commodities
Other
Operational efficiency in all business lines:
Corporate finance
Trading & sales
Retail banking
Commercial banking
Payment & settlement
Agency services & custody
Asset management
Retail brokerage
Commodities & Equities
Short- and long-term business strategies
Transactions
Other

Globalization

- Increasing globalization in the activities of banks.
- Wider range of financial products and services.
- Increasing levels of financial risk exposures, including market risk.

Bank of International Settlements (BIS)

- Basel, Switzerland
- Established on 17 May 1930
- The world's oldest international financial organization.
- **An organization of central banks with 58 member central banks including the BSP**.
- Also operates in the private market with central banks and international financial institutions as clients.

The Bangko Sentral ng Pilipinas(BSP)

- Has **supervision over the operations of local banks** (universal banks, commercial banks, savings banks, thrift banks, rural banks)
- Exercises **regulatory powers over the operations of finance companies** and non-bank financial institutions performing quasi-banking functions.

Basel I

- Published in 1988
- Set of **rules on bank capital requirements for credit risk**.
- 1996 **amendment** - **new capital requirement for market risk** was published. Implemented in 1998.

CIRCULAR NO. 280
As amended
Series of 2001
The Monetary Board in its Resolution No. 285 dated 16 February 2001, approved the following **guidelines on the adoption in the Philippines of the risk-based capital adequacy framework** pursuant to Section 34 of The General Banking Law of 2000. **The guidelines shall initially cover only capital requirements for credit risks** pending issuance of supplementary guidelines to incorporate market risks. Upon effectivity, the guidelines shall replace the existing provisions of Section X116 and its subsections of the Manual of Regulations for Banks, which are hereby revised to read, as follows:

"**Sec. X116 Minimum Ratio. The risk-based capital ratio of a bank, expressed as a percentage of qualifying capital to risk-weighted assets, shall not be less than ten percent (10%)** for both solo basis (head office plus branches) and consolidated basis (parent bank plus subsidiary financial allied undertakings, but excluding insurance companies).
The ratio shall be maintained daily.

“§ X116.2 Risk-Weighted Assets. The risk-weighted assets shall be determined by

assigning
risk weights to amounts of on-balance sheet assets and to credit equivalent amounts of off-balance
sheet items (inclusive of derivative contracts): *Provided, That the following shall be* deducted from the total risk-weighted assets:

1. General loan loss provision (in excess of the amount permitted to be included in upper Tier 2 capital); and
2. Unbooked valuation reserves and other capital adjustments affecting asset accounts based on the latest report of examination as approved by the Monetary Board.

a. For on-balance sheet assets, the risk-weighted amount shall be the product of the book value of asset multiplied by the risk weight associated with that asset,

Date Issued: 12.03.2002

CIRCULAR NO. 360
Series of 2002

Pursuant to Monetary Board Resolution No. 1616 dated November 7, 2002, **approving the guidelines to incorporate market risk in the risk-based capital adequacy framework for universal banks and commercial banks,** the provisions of the Manual of Regulations for Banks are amended as follows:

1. A new subsection is hereby added after Subsection X116.4 of the Manual to read as follows:

"§ I116.5. Market Risk Capital Requirement. Universal banks (UBs)/ commercial banks (KBs) shall also measure and apply capital charges for market risk, in addition to the credit risk capital requirement described above, in accordance with the Guidelines to Incorporate Market Risk in the Risk-Based Capital Adequacy Framework (Appendix __)."

Basel II

- 1999 – significant changes in the proposed calculation of proposed capital requirements for credit risk.
- 1999 – introduction of capital requirement for operational risk.

Basel II – Underlying Principles

1. Pillar 1 – Minimum Capital Requirements.

   Banks should have capital appropriate for their risk-taking activities.

2. Pillar 2 – Supervisory Review Process.

   Banks should be able to properly assess their capital adequacy in relation to the risk they are taking, and supervisors should be able to evaluate the soundness of these assessments.

3. Pillar 3 - Market Discipline

   Banks should be disclosing pertinent information necessary to enable market mechanism to complement the supervisory oversight function.

CIRCULAR NO. 538
Series of 2006

Subject: **Revised Risk-Based Capital Adequacy Framework**

The Monetary Board, in its Resolution No. 697 dated 2 June 2006, approved the attached guidelines implementing the revised risk-based capital adequacy framework for the Philippine banking system to conform to Basel II recommendations.

The guidelines apply to all universal banks and commercial banks, as well as their subsidiary banks and quasi-banks. Thrift banks, rural banks, as well as quasi-banks that are not subsidiaries of universal banks and commercial banks shall continue to be subject to the existing applicable risk-based capital adequacy framework, as contained in Circular No. 280 dated 29 March 2001, as amended, and Circular No. 400 dated 1 September 2003.

The appropriate supervisory reporting template to implement the revised framework shall be issued at a later date. Such reports shall be submitted quarterly, and shall be classified under Category A-1 Reports.

This Circular shall take effect on 1 July 2007.

CIRCULAR NO. 639
Series of 2009

Subject: **Internal Capital Adequacy Assessment Process (ICAAP) and Supervisory Review Process (SRP)**

The Monetary Board in its Resolution No. 1684 dated 19 December 2008 approved the attached guidelines on banks' internal capital adequacy assessment process (ICAAP) and the BSP's supervisory review process (SRP).

The ICAAP guidelines shall apply to all universal banks and commercial banks on a group-wide basis.

Covered banks shall submit a trial "ICAAP Document" to the appropriate Central Point of Contact Department (CPCD) of the BSP on 31 March 2009.

The attached guidelines shall take effect on 1 January 2010.

Revisions in the Basel II Market Risk Framework

- After the 2007 credit crunch
- Key elements
  - incremental risk capital charge for credit-sensitive positions, including securitization activities.
  - Stressed-tested Value at Risk (stressed VaR)
- Leading to Basel III

VaR in Banking Regulation

- 1996 amendment distinguished between banking book and trading book.
- Banking book for loans.
- Trading book for traded instruments (stocks, bonds, swaps, forwards, options etc)
- **1996 amendment calculated required capital for the trading book using VaR**.

VaR in Banking Regulation

- VaR is revaluation loss over N days (e.g. 10 days that is expected to be exceeded only 1% of the time
- Before the 2007 crisis, **the required capital for market risk is**

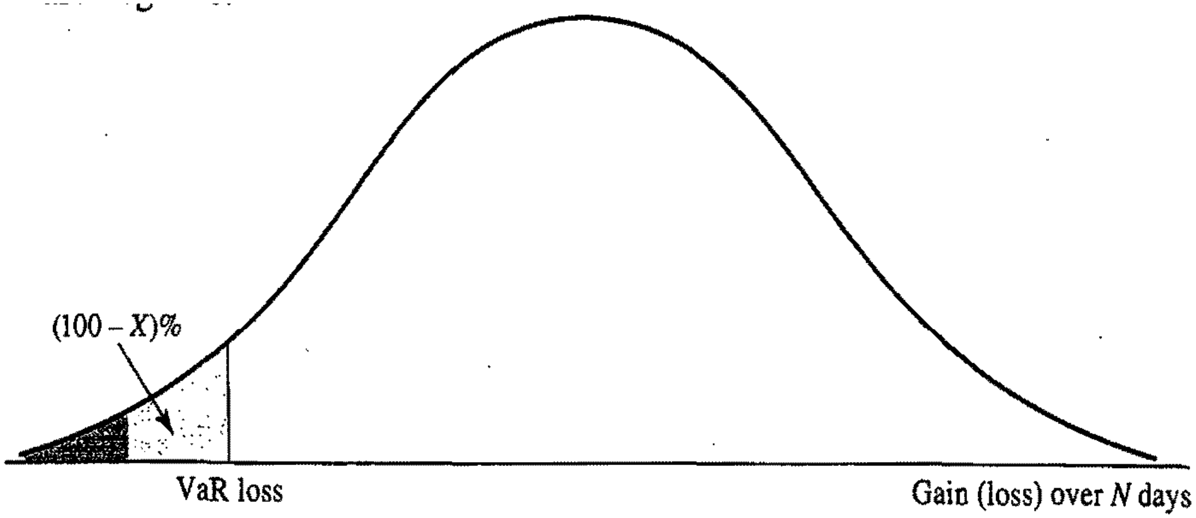
$$k * VaR$$

- The rules were **revised after 2007**.

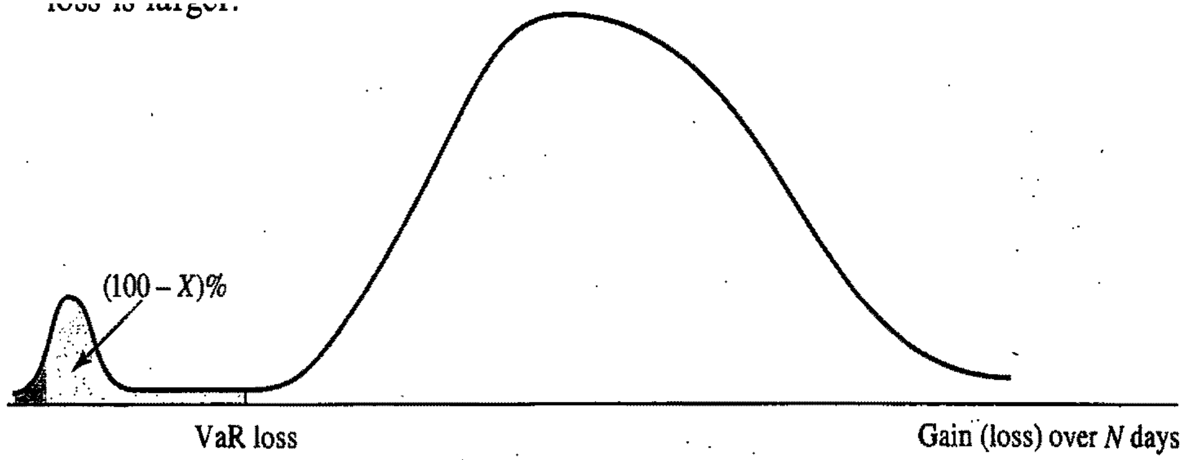
VaR in Risk Management

- How bad can things get in the next N days?
- What is the N-day portfolio revaluation loss level that has only 1% chance of getting exceeded?

VaR



VaR



Portfolio Market Value

- The sum of the market values (in domestic currency) of all tradable assets in the portfolio. Among these are:
  - Stocks
  - Debt securities(e.g. Treasury or Corporate bonds)
  - Foreign currency
  - commodities
  - Derivatives

Portfolio Risk Factors

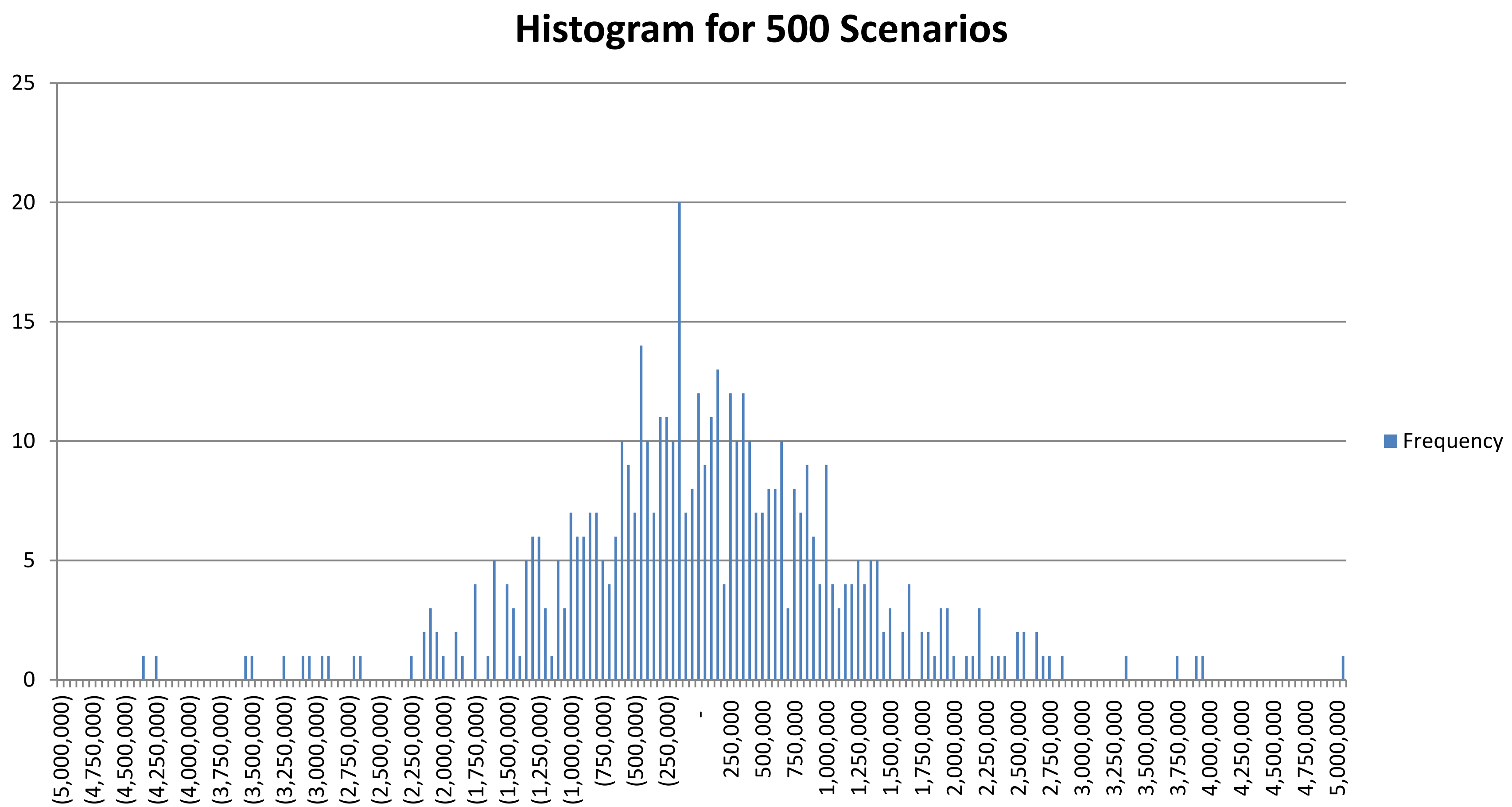
- Stock or equity price
- Interest or yield rates
- Foreign exchange rates
- Commodity prices
- Etc.

➢ Each risk factor may have its own separate model calibrated against historical data.

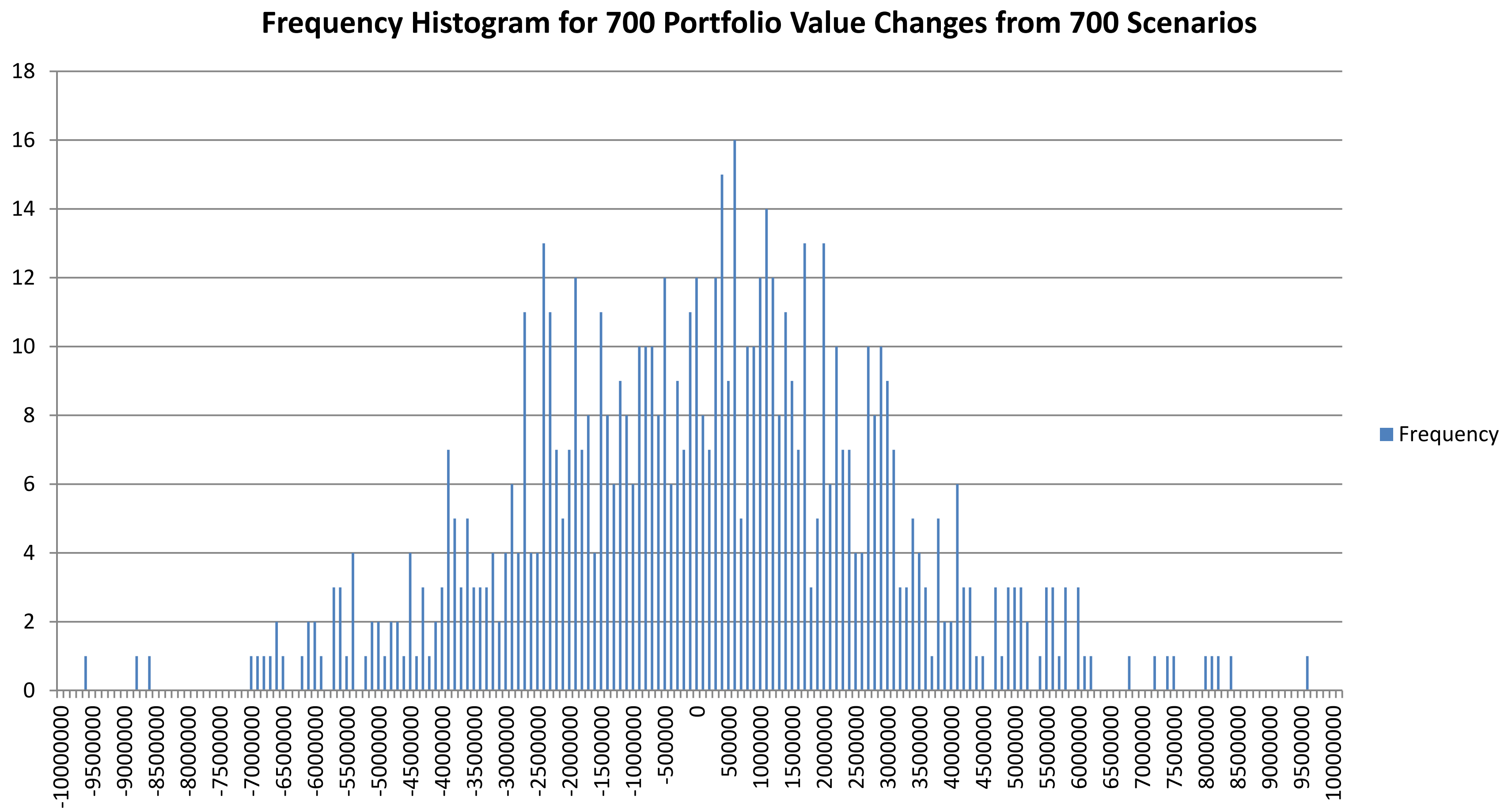
VaR from Historical Simulation

- Basel II requires a historical observation period of at least one year.
- **What happened in the past m days provides m equally likely scenarios for what might happen between today and tomorrow**.
- Two examples
  - Two-stock portfolio
  - Forex portfolio

Example 1: Historical Simulation



Example 2: Forex VaR



Historical Simulation

- Strengths
  - Simple and easy to implement.
  - No model parameters to estimate.
- Weaknesses
  - Computationally slow.
  - Volatility of risk factors not updated.

Model Building Approach

- Assumptions:
  - Model for the risk factors
  - Risk Factor returns are normal with mean 0.
  - daily volatility of a risk factor is the variance of the daily factor return.

Single Asset Portfolio

- A model for the asset price: $\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$
- Stock price return: $\frac{dS_t}{S_t} \sim N(0, \sigma^2 dt)$
- Daily volatility: $\sigma_1 = \sigma\sqrt{dt}, \;\; dt = \frac{1}{252}$

Single Asset Portfolio

- The 1-day 99% VaR today is $|V| = \sigma_1 \left|\alpha N^{-1}(0.01)\right|$
- Here, α is the position (short or long, in units of the domestic currency).
- **Estimating VaR involves estimating σ_1 from historical data of stock prices.** .

A Simple Example

- Today's closing price of stock A is P25.45. Historical data estimates its volatility to be 26% p.a.. An investor holds 5M shares of the stock. Using the model, the investor is 99% confident that the value of his investment at the end of tomorrow's trading day will not be lower than

$$127,250,000 - \left(127,250,000 * \frac{0.26}{\sqrt{252}} * |N^{-1}(0.01)|\right)$$

- i.e. a 1-day 99% VaR of P484,848 .
- The 10-day 99% VaR is P1,533,223.71

Two-Asset Portfolio

- Two risk factors S_1 and S_2
- Two positions α_1, $\alpha_{2,}$ daily volatilities σ_1, σ_2.
- Vector of daily factor returns

$$(R_1, R_2)^T \sim N(\mathbf{0}, \Sigma)$$

- where

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{bmatrix}$$

- and ρ is the correlation between the returns.

Two-Asset Portfolio

- It follows that for i = 1, 2, $R_i \sim N(0, \sigma_i^2)$.
- Change in portfolio value is $\Delta P = \alpha_1 R_1 + \alpha_2 R_2$.
- With distribution

$$\Delta P \sim N(0, \alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2)$$

Two-Asset Portfolio

- Let $\sigma_P = \sqrt{\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2}$

- **The benefits of diversification**

$$\sigma_P = \sqrt{(\alpha_1\sigma_1 + \alpha_2\sigma_2)^2 - 2(1-\rho)\alpha_1\alpha_2\sigma_1\sigma_2}$$

if $\rho < 1$, we have

$$\sigma_P < \sigma_1|\alpha_1| + \sigma_2|\alpha_2|$$

Two-Asset Portfolio

- A single-asset portfolio consisting of an asset with price volatility σ_i and position α_i would have a 1-day, 99% VaR of $\sigma_i \left|\alpha_i N^{-1}(0.01)\right|$.

- A two-asset portfolio would have a 1-day 99% VaR of $\sigma_P \left|N^{-1}(0.01)\right|$,

- which is smaller than $(\sigma_1|\alpha_1| + \sigma_2|\alpha_2|) \left|N^{-1}(0.01)\right|$

Two-Asset Portfolio

- If we let $\alpha = (\alpha_1 \ \alpha_2)^T$ and $\mathbf{R} = (R_1 \ R_2)^T$ then
- we have $\Delta P = \alpha^T \mathbf{R}$ and
- the variance can be written as $\sigma_P^2 = \alpha^T \Sigma \alpha$.
- Alternatively, $\sigma_P^2 = [\alpha_1\sigma_1 \ \alpha_2\sigma_2] \, \tilde{\rho} \, [\alpha_1\sigma_1 \ \alpha_2\sigma_2]^T$
- where $\tilde{\rho}$ is the correlation matrix

$$\tilde{\rho} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The Linear Model (n-Asset Portfolio)

- Consider an n-asset portfolio consisting of investment positions in stocks, stock indices, bonds, commodities and foreign currencies.

- Portfolio is valued in terms of the domestic currency.

- VaR is measured in terms of the domestic currency.

The Linear Model (n-Asset Portfolio)

- We may consider the risk factors S_i to be the market values of the included stocks, stock indices, bonds and foreign currencies.

- Let R_i be the daily return on the i^{th} risk factor.

- The vector of daily returns, $\mathbf{R} = (R_1\ R_2\ \dots\ R_n)^T$ is assumed to be multivariate normal with

$$\mathbf{R} \sim N(\mathbf{0}, \Sigma)$$

The Linear Model (n-Asset Portfolio)

- Let the position vector be $\alpha = (\alpha_1 \; \alpha_2 \; \ldots \; \alpha_n)^T$.
- Then $\Delta P = \alpha^{\mathbf{T}}\mathbf{R}$ with $\Delta P \sim N(0, \alpha^{\mathbf{T}}\Sigma\alpha)$.
- Let $\sigma_P = \sqrt{\alpha^{\mathbf{T}}\Sigma\alpha}$,
- We may also write $\sigma_P = \alpha^{\mathbf{T}}\sigma\tilde{\rho}\alpha\sigma\alpha$,
- where

$$\sigma = \begin{bmatrix} \sigma_1 & 0 & \ldots & 0 \\ 0 & \sigma_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_n \end{bmatrix}$$

The Linear Model (n-Asset Portfolio)

- So the 1-day, 100c% VaR for this portfolio is

$$\sigma_P \left| N^{-1}(1-c)) \right|$$

- **The real challenge is to get fairly good estimates of the correlations and standard deviations of the returns from historical data**.

- Limitation: Factor returns may not be normally distributed. This assumption may need to be tested statistically.

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Treasury Bills and Bonds

- An **T-zero coupon bond** is an investment that provides a certain cash flow C after T years (the maturity).

- Issued by the government through the central bank to borrow it its own currency.

- Considered riskless.

- The **T-year yield** today is the discounting rate that determines the market price B of this bond today.

Treasury Bills and Bonds

- The **T-year risk-free rate** may be considered to be the T-year yield on an T-year zero coupon bond.

- The T-year risk-free rate **r(T) expressed with compounding frequency m per year** is the solution to the equation

$$B\left(1+\frac{r}{m}\right)^{mT} = C$$

Treasury Bills and Bonds

- When **expressed with continuous compounding,** r(T) is given by $Be^{rT} = C$
- Standard maturities sold in the market are 3m, 6m, 1y, 2y, 3y, 4y, 5y, 7y, 10, 20y, 25y.
- The **yield curve** is the graph of r as a function of the maturity T, where T may take values other the standard maturities. (Market Information about this yield curve is only at a discrete set of points – the standard maturities)

Treasury Bills and Bonds

- A **coupon bond** provides cash flows at some points before maturity aside from the cash flow provided at maturity.

- A coupon bond providing cash flows c_1, c_2, …, c_n at times t_1, t_2, …, t_n, respectively may be regarded as a portfolio of n zero-coupon bonds with maturities t_1, t_2, …, t_n.

Treasury Bills and Bonds

- A single T-year zero-coupon bond with current price B and yield y (with continuous compounding) would satisfy

$$B = Ce^{-yT}$$

- The percentage change in the bond price (one-day return) in the bond price tomorrow is given by

$$\frac{\Delta B}{B} \approx T\Delta y$$

Treasury Bills and Bonds

- So the factor return $R = \frac{\Delta B}{B} \sim N(0, T^2\sigma_1^2)$ if

$$\Delta y \sim N\left(0, \sigma_1^2\right)$$

- The 1-day, 99% VaR of one such bond is about

$$BT\sigma_1 2.326$$

Treasury Bills and Bonds

- For n zero coupon bonds with present prices $B_1, B_2, \ldots, B_n$ and standard maturities $T_1, T_2, \ldots, T_n$, the new bond prices tomorrow are determined by the yield curve tomorrow.

- The change in portfolio value is then,

$$\Delta P \approx B_1 T_1 \Delta y_1 + B_2 T_2 \Delta y_2 + \ldots + B_n T_n \Delta y_n.$$

VaR for Bonds : Duration Method

- Let y be the yield (**IRR**) on the bond portfolio, i.e. the number y satisfying

$$P = C_1 e^{-yT_1} + C_2 e^{-yT_2} + \ldots + C_n e^{-yT_n}$$
$$D = w_1 T_1 + w_2 T_2 + \ldots w_n T_n$$
$$w_j = \frac{C_j e^{-yT_j}}{\sum_{i=1}^n C_i e^{-yT_i}} \qquad \Delta P = -PD\Delta y$$

- Let Δy be the daily movement in this yield rate with $\sigma_1 = Var(\Delta y)$. The 1-day 99% portfolio VaR is $PD\sigma_1 N^{-1}(0.01)$.

VaR for Bonds : Duration Method

- Duration method often gives inaccurate estimates of the VaR for large movements in the yield rate.
- Assumes that daily moves in the interest rates is driven by a single risk factor.

VaR for Bonds : Cash Flow Mapping

- Consider a cash flow C due on date T between two standard maturities T_i and T_{i+1}. Let the corresponding zero-coupon yield rates be y_i and y_{i+1}, respectively.

- The idea is to map this single cash flow into two positive cash flows C_i and C_{i+1} at dates T_i and T_{i+1}, respectively so that the following hold:

VaR for Bonds : Cash Flow Mapping

- Present value of the cash flow is preserved:

$$Ce^{-yT} = C_i e^{-y_i T_i} + C_{i+1} e^{-y_{i+1} T_{i+1}}$$

where y is linearly interpolated between known market rates y_i and y_{i+1}

- Variance (of the present value) of the single cash flow is preserved:

$$(B\sigma)^2 = [\ B_i \quad B_{i+1}]\ ] \left[\begin{array}{cc} \sigma_i^2 & \rho\sigma_i\sigma_{i+1} \\ \rho\sigma_i\sigma_{i+1} & \sigma_{i+1}^2 \end{array} \right] \left[\begin{array}{c} B_i \\ B_{i+1} \end{array} \right]$$

where σ is the linearly interpolated from daily bond price volatilities σ_i and σ_{i+1} .

VaR for Bonds : Cash Flow Mapping

- The result of cash flow mapping is that any bond (coupon-bearing or not) is transformed into into some portfolio of n zero coupon bonds B_1, B_2, …, B_n with standard maturities T_1, T_2, …, T_n.
- The change in the value of this portfolio is given by $B_1 R_1 + B_2 R_2 + \ldots + B_n R_n$ where the daily price returns are $$R_i = \frac{\Delta B_i}{B_i}$$

VaR for Bonds : Cash Flow Mapping

- Using the linear model, we have

$$\sigma_P = \mathbf{B}^T \sigma \tilde{\rho} \sigma \mathbf{B}$$

where $\mathbf{B} = \left[\begin{array}{cccc} B_1 & B_2 & \ldots & B_n \end{array} \right]$

- the 1-day 99% VaR is $\sigma_P N^{-1}(0.01)$

VaR for Bonds:
Principal Component Analysis

- Recall that for a portfolio P of n zero-bonds with standard maturities, we have

$$\Delta P \approx B_1 T_1 \Delta y_1 + B_2 T_2 \Delta y_2 + \ldots + B_n T_n \Delta y_n$$

- The daily interest rate movements Δy_i are n random variables that may be highly correlated with one another.

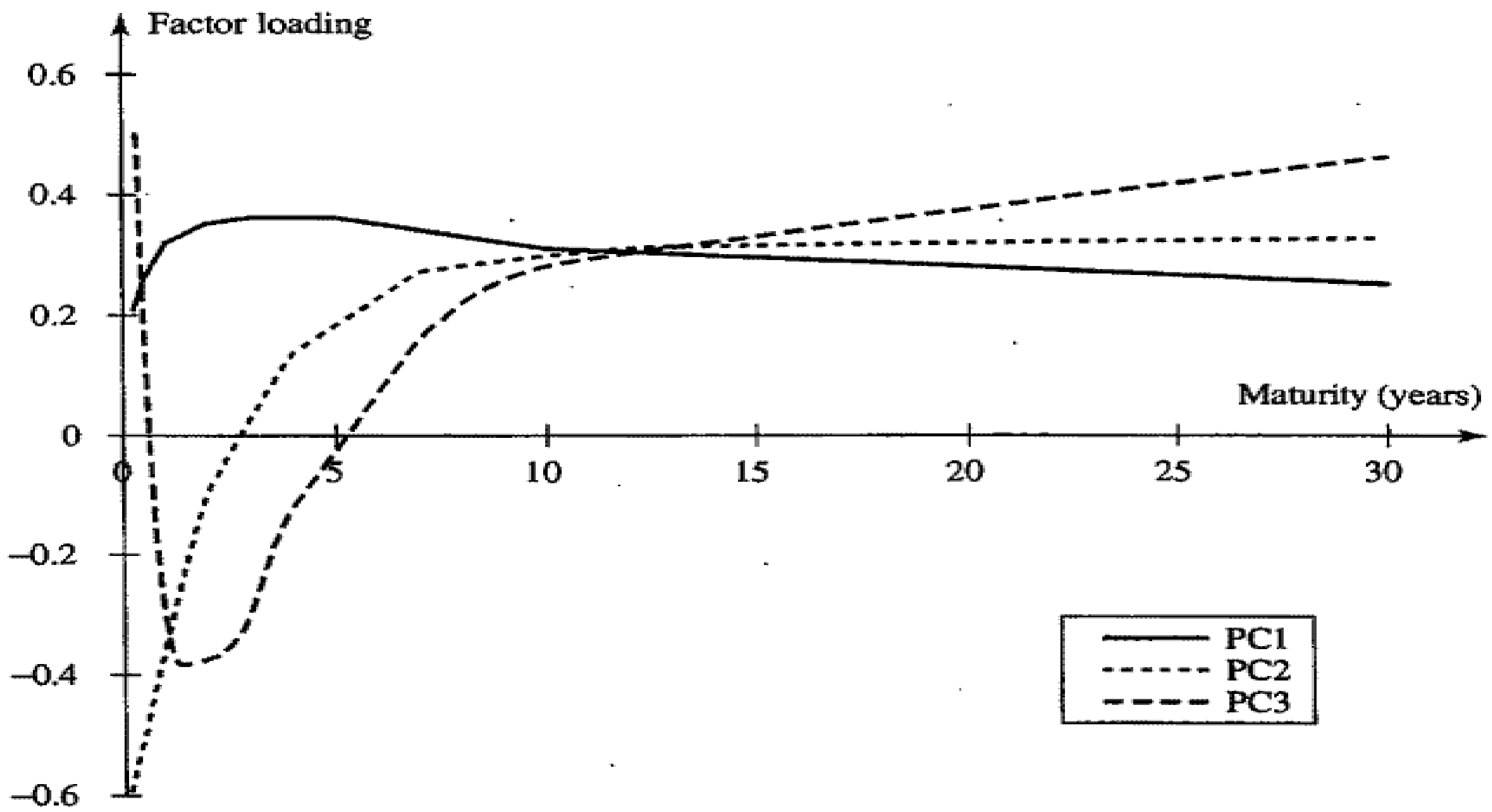
VaR for Bonds:
Principal Component Analysis

	PC1	*PC2*	*PC3*	*PC4*	*PC5*	*PC6*	*PC7*	*PC8*	*PC9*	*PC10*
3m	0.21	−0.57	0.50	0.47	−0.39	−0.02	0.01	0.00	0.01	0.00
6m	0.26	−0.49	0.23	−0.37	0.70	0.01	−0.04	−0.02	−0.01	0.00
12m	0.32	−0.32	−0.37	−0.58	−0.52	−0.23	−0.04	−0.05	0.00	0.01
2y	0.35	−0.10	−0.38	0.17	0.04	0.59	0.56	0.12	−0.12	−0.05
3y	0.36	0.02	−0.30	0.27	0.07	0.24	−0.79	0.00	−0.09	−0.00
4y	0.36	0.14	−0.12	0.25	0.16	−0.63	0.15	0.55	−0.14	−0.08
5y	0.36	0.17	−0.04	0.14	0.08	−0.10	0.09	−0.26	0.71	0.48
7y	0.34	0.27	0.15	0.01	0.00	−0.12	0.13	−0.54	0.00	−0.68
10y	0.31	0.30	0.28	−0.10	−0.06	0.01	0.03	−0.23	−0.63	0.52
30y	0.25	0.33	0.46	−0.34	−0.18	0.33	−0.09	0.52	0.26	−0.13

Source: Hull, J. 6th ed.

VaR for Bonds:
Principal Component Analysis

PC1	*PC2*	*PC3*	*PC4*	*PC5*	*PC6*	*PC7*	*PC8*	*PC9*	*PC10*
17.49	6.05	3.10	2.17	1.97	1.69	1.27	1.24	0.80	0.79



Other VaR Methods

1. Quadratic Model
   - Used for a portfolio that contains options

$$dP \approx \delta^T \mathbf{R} + \frac{1}{2}\mathbf{R}^T \Gamma \mathbf{R}$$

2. Monte Carlo Simulation
   a) Full Monte Carlo
   b) Partial Monte Carlo
   - Sampling of the risk factor returns from a multivariate normal distribution
   - Computationally expensive.

Using VaR

- Banks in Europe and in the US have used many of the methods described.
- Local banks have started to using these methods only recently.
- Locally, historical simulation and the linear model are the most commonly used methods.

Back-testing

- Reality check.
- Testing how well the VaR method would have performed in the past (normal conditions).
- Review the method based on performance from back-testing.

Stress-testing

- Estimating a portfolio's performance under scenarios of extreme market movements.

- Setting the returns in market variables equal to those seen in some extreme market moves.

- After 2007, regulators proposed calculation of stressed VaR on a regular basis.

Other Risk Measures

- Expected shortfall or TVaR.
  - E(X|X>VaR)
- Coherent Measures of Risk: Let X, Y be loss random variables, ρ is a coherent measure of risk if it satisfies the following:
  1. Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
  2. Monotonicity: If $X \leq Y$ then $\rho(X) \leq \rho(Y)$
  3. Positive homogeneity: If c >0, then ρ(cX) = cρ(X).
  4. Translation invariance: If c > 0, then ρ(X+c) =ρ(X)+c

Research Directions

VaR estimation for derivatives that can be considered as a portfolio of bonds.

1. Estimating the yield curve.
   - Nelson-Siegel model applied on Philippine data
   - Cox-Ingersoll-Ross model of the short rate
2. Forecasting the yield curve
   - Nelson-Siegel model
   - Method of copulas

Research Directions

VaR estimation for derivatives that can be considered as a portfolio of bonds.

1. Using forecasted yield curve
2. Using quadratic model
3. Using Ito’s lemma

THANK YOU!

HAVE A NICE DAY!